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| Related image | **KONERU LAKSHMAIAH EDUCATION FOUNDATION**  (Deemed to be University estd, u/s, 3 of the UGC Act, 1956) (NAAC Accredited “A++” Grade University)  Green Fields, Guntur District, A.P., India – 522502  **Department of Computer Science and Engineering**  (DST - FIST Sponsored Department) |  |

**B.Tech. II CSE(H) PROGRAM**

**A.Y. 2023-24 ODD, Semester-II**

**Course Code: 22MT2005**

**PROBABILIRT, STATISTICS AND QUEUING THEORY**

**Course Outcome-1**

**Session 4:** **Conditional Probabilities - Independent Events, Bayes Formula.**

**Course Description (Description about the subject)**

1. **Aim**

To explain the rules of conditional probability and independence of events

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1. **Instructional** **Objectives (Course Objectives)**

Demonstrate the concept of conditional probability with examples, List out the rules of dependent and independent events, Describe the Bayes rule, and Discuss the importance of the Bayes rule and its applications.

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1. **Learning** **Outcomes (Course Outcome)**
2. **CO1**: Students will be able to explain conditional probability, Describe the rules of dependent and independent events, and summarize the concepts of Bayes rule and its applications.

**Module** **Description** **(CO-1 Description)**

Bayes' theorem allows us to calculate the probability of a hypothesis being true given the

occurrence of some observed evidence.

1. **Session** **Introduction**

In this lesson, we will look at conditional probabilities, comprehend the idea of event independence, and examine the Bayes formula. In order to make better decisions when dealing with unclear information, we will learn how to compute conditional probabilities, evaluate the results, and apply Bayes' theorem. You ought to have a firm grasp on these foundational ideas in probability as well as how they apply in real-world situations by the end of this lesson.

1. **Session description**

In this lesson, you will gain a thorough grasp of conditional probabilities, the independence of events, and how to apply Bayes' formula to make educated choices in ambiguous circumstances. This webinar will give you useful methods for managing uncertainty and coming to intelligent decisions, whether you're a data scientist, a researcher, or just someone who wants to get better at probabilistic reasoning. Join me as I take you on an interesting voyage into the world of conditional probabilities and Bayes' theorem.

Conditional probability:

The probability of happening of an event ‘A when the event ‘B’ has already happened is called conditional probability of A/B.

P(A/B)=, P(B)

The probability of happening of an event ‘B when the event ‘A’ has already happened is called conditional probability of B/A.

P(B/A) =, P(A)

Conditional probability can be contrasted with unconditional probability. Unconditional probability refers to the likelihood that an event will take place irrespective of whether any other events have taken place or any other conditions are present.

One of the objectives of calculating conditional probability is to determine whether two events are related.

**Dependent events**

Two events are dependent when the outcome of the first event influences the outcome of the second event . An event is deemed dependent if it provides information about another event. An event is deemed independent if it offers no information about other events.

If A and B are dependent events then the probability of both occurring is

P(AՈB)=P(A).P(B/A)

P(AՈB)=P(B).P(A/B)

**Example:**

1.Boarding a plane first and finding a good seat

2.Getting into a traffic accident is dependent upon driving or riding in a vehicle.

**MULTIPLICATIVE RULE**

If in a experiment the events A and B can both occur, then P(A)provided P(A) > 0. We can also write )= P(B) in other words, it does not matter which event is referred to as A and which event is refered to as B.

Note:

1) Two events A and B are independent if and only if P(AՈB)=P(A)P(B).

Eg: :Let A be the event that raw material is available when needed and B be the event that the matching time is less than one had. If P(A)=0.8 and P(B)=0.7. What is P(AՈB).

2) If A, B, and C are any three events then the multiplicative rule

P(AՈBՈC)= P(A) P(B/A) P(C/AՈB)

3) P(T G P)=P(T).P(I/T).P(G/ T I )P(P/ T G)

3) If A, B, and C are independent events if and only if

P(AՈBՈC)=P(A)P(B)P(C)

**Example 1:** In a group of 100 computer buyers, 40 bought CPU, 30 purchased monitor, and 20 purchased CPU and monitors. If a computer buyer chose at random and bought a CPU, what is the probability they also bought a Monitor?

**Solution:** As per the first event, 40 out of 100 bought CPU, So, P(A) = 40% or 0.4

Now, according to the question, 20 buyers purchased both CPU and monitors.

So, this is the intersection of the happening of two events. Hence, P(A∩B) = 20% or 0.2

By the formula of conditional probability, we know;

P(B|A) = P(A∩B)/P(A)

P(B|A) = 0.2/0.4 = 2/4 = ½ = 0.5

The probability that a buyer bought a monitor, given that they purchased a CPU, is 50%.

**Example 2:**

In a batch, there are 80% C programmers, and 40% are Java and C programmers. What is the probability that a C programmer is also Java programmer?

Let A --> Event that a student is Java programmer

B --> Event that a student is C programmer

P(A|B) = P(A ∩ B) / P(B) = (0.4) / (0.8) = 0.5

So there are 50% chances that student that knows C also knows Java.

**Example 3:** The chance of a student passing an exam is 20%. The chance of passing an exam and getting above 90% marks in it is 5% given that the student passes the examination,the probability that the student gets above 90% marks is [2015 Mech set 2]

A ) 1/18 B) 1/4 C) 2/9 D) 5/18

**Solution:** Let A and B denote the events of a student passing an exam and a student getting above 90% marks in the exam respectively.

P(A)=20/100=0.2, P(AՈB)=5/100=0.05.

Given that a student passes the examination, the probability that the students gets above 90% marks

=P(B/A)= P(AՈB)/P(A)= 0.05/0.2=1/4.

Example 2: P(X) = 1/4, P(Y) = 1/3, P(X) = 1/12 The value of P(Y/X) is [2015 Mech set 3]

1. 1/4 B) 4/25 C) 1/3 D) 29/50

P(Y/X)= P(YՈX)/P(X)= 1/12/1/4=1/3.

**TOTAL PROBABILITY**

If the events *B1,B2,…,BK* constitute a partition of the sample space S such that



then for any event A of S,



**Example: 1** A group contains equal no of men and women of those 20 % of the men, 50 % of women are unemployed. If a person is selected at random from these. The probability of the selected person being employed is ----- [2014 mech]

**Solution:**

Let E be the event that the probability of the selected person being employed is

If a man is selected, then the probability of him being employed=20/100.

Similarly, if the woman was selected the probability of her being employed=50/100

So probability=P(M).P(E/M)+P(W).P(E/W)=(1/2)\*(20/100)+(1/2)\*(50/100)=(1/10)+(5/20)=7/20.

Required probability =1-P(E)=1-0.35=0.65.

**Bayes Rule**

Hypotheses: The events E1 , E2 ,… En is called the hypotheses

Priori Probability: The probability P(Ei ) is considered as the priori probability of hypothesis Ei

Likelihood Probability: The Probability of P(A/Ei) is considered as the event likely to occur.

Posteriori Probability: The probability P(Ei |A) is considered as the posteriori probability of hypothesis Ei

Bayes’ theorem is also called the formula for the probability of “causes”.

Since the Ei ‘s are a partition of the sample space S, one and only one of the events Ei occurs (i.e. one of the events Ei must occur and the only one can occur). Hence, the above formula gives us the probability of a particular Ei (i.e. a “Cause”), given that the event A has occurred.

**If the events B1, B2,…,Bk constitute a partition of the sample space S such that P(Bi in S such that P(A)**

**One of the many applications of Bayes’ theorem is Bayesian inference, a particular approach to statistical inference. Bayesian inference has found application in various activities, including medicine, science, philosophy, engineering, sports, law, etc.**

**Example: Bayes’ theorem to define the accuracy of medical test results by considering how likely any given person is to have a disease and the test’s overall accuracy.**

**Bayes’ theorem relies on consolidating prior, Likelihood probability distributions to generate posterior probabilities. In Bayesian statistical inference, prior probability is the probability of an event before new data is collected.**

**Example l:** The probability that student knows the correct answer to a multiple-choice question is 2/3. If the student does not know the answer, the student guesses the answer. The probability of guessed answer is correct is ¼ . Given that student has answered the question correctly. The conditional probability that student knows the correct answer is [2013 mech]

1. 2/3 B) 3/4 C) 5/6 D) 8/9

**Example 2:**

Given the following statistics, what is the probability that a woman has cancer if she has a positive mammogram result?

1. One percent of women over 50 have breast cancer.
2. 90% of women who have breast cancer test positive on mammograms.
3. 8 percent of women will have false positives.

**Step 1:** Assign events to A or X. You want to know what a woman’s probability of having cancer is, given a positive mammogram. For this problem, actually having cancer is A and a positive test result is X.

**Step 2:** List out the parts of the equation (this makes it easier to work the actual equation): P(A)=0.01 P(~A)=0.99 P(X|A)=0.9 P(X|~A)=0.08

**Step 3:** Insert the parts into the equation and solve. Note that as this is a medical test, we’re using the form of the equation from example #2: (0.9 \* 0.01) / ((0.9 \* 0.01) + (0.08 \* 0.99) = 0.10.

The probability of a woman having cancer, given a positive test result, is 10%

1. **Activities/ Case studies/related to the session.**

**Independent events**

You toss a coin and it comes up "Heads" three times ... what is the chance that the next toss will also be a "Head"? The chance is simply ½ (or 0.5) just like ANY toss of the coin.

What it did in the past will not affect the current toss! Some people think "it is overdue for a Tail", but really truly the next toss of the coin is totally independent of any previous tosses.  **A diagram of a diagram of a number of circles

Description automatically generated with medium confidence**

1. **Examples & contemporary extracts of articles/ practices to convey the idea of the Session**
2. **SAQ's-Self Assessment Questions**

1. Let P(E) denote the probability of the event E. Given P(A) = 1, P(B) = 1/2, the values of P(A | B) and P(B | A) respectively are:

2. Two fair dice are thrown. What is the probability of the sum of 10 being obtained for the two uppermost faces ?

1. **Summary**

In this session, the concept of conditional probability, Applications of Baye’s rule have discussed.

1. How to find conditional probability.

2. Difference between Dependent and independent events

3. State the importance of Baye’s Rule

4. Baye’s Rule and its applications.

1. **Terminal Questions**

1.Describe in detail about the dependent and independent events

2. List out different type of probabilities used in Baye’s Rule

3. How do you find the conditional probability.

4. Summarize the Baye’s rule and its applications

**5.** A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that

a) Both the ambulance and the fire engine will be available.

b) Ambulance or fire engine available.

6.The odds that a book will be reviewed favorably by three independent critics are 5 to 2, 4 to 3, and 3 to 4. Find the probability that of the three reviews, a majority will be favorable.

7. The Probability that a regularly scheduled flight departs on time is P(D)=0.83; the probability that it arrives on time is P(A)=0.82; and the probability that it departs and arrives on time is P(DՈA)=0.78

Find the probability that a plane

(a) arrives on time given that it departed on time.

(b) departed on time given that it has arrived on time,

(c) neither departed on time nor arrived on time.

8. Amy commutes to work by two different routes A and B. If she comes home by route A, then she will be home no later than 6 P. M. with probability 0.8, but if she comes home by route B, then she will be home no later than 6 P. M. with probability 0.7. In the past, the proportion of times that Amy chose route A is 0.4. If Amy is home after 6 P. M. today, what is the probability that she took route B?

9. Two firms V and W consider bidding on a road building job, which may or may not be awarded depending on the amounts of the bids. Firm V submits a bid and the probability is 3/4 that it will get the job provided firm W does not bid. The probability is 3/4 that W will bid, and if it does, the probability that V will get the job is only 1/3.

1. What is the probability that V will get the job?
2. If V gets the job, what is the probability that W did not bid?
3. **Case Studies (CO Wise)**

**NA**

1. **Answer Key**

**NA**

1. **Glossary**

**NA**

1. **References of books, sites, links Textbooks:**

**Textbooks:**

1. Probability and Statistics Rukmangad Achari E. and E. Keshava Reddy
2. Probability and Statistics for Engineers and Scientists” Ronald E. Walpole, Sharon L. Myers and Keying Ye 8th Edition Pearson pub
3. Probability & Statistics for Engineers Dr. J. Ravichandran first Edition Wiley-India

**Reference books:**

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund’s Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

**Web Resources**

1. \* https://ncert.nic.in/textbook.php?kemh1=16-16 \*
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf

3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5\_MITRES\_6\_012S18\_L01.pd f 9

1. **Keywords**

Conditional Probabilities , Independent Events, Bayes Formula